## SPACE, TIME, AND GRAVITATION[1]

### By Professor EDWIN B. WILSON
#### MASSACHUSETTS INSTITUTE OF TECHNOLOGY

1. *Unrest in Physical Science.*—What you ask me to perform in presenting to you some discussion of the new conception of the categories of space and time suggested by Einstein's treatment of universal gravitation is a task that I accept reluctantly, even among friends and philosophers, because the matter is so new that an adequate judgment can no more be given to-day than a satisfactory judgment upon Maxwell's theory, whether in its philosophical or its physical significances, could have been given in the early days when he was forming it in his memoirs and before the appearance of his great treatise.

There is moreover, to-day, in the physical world a general unrest, little realized by non-physicists, and quite unlike the condition, I believe, existing fifty years ago. This unrest leads physicists to alternate, according to their temper, between a despair of ever settling the physical bases of the many new facts which experiment is thrusting upon them and a desperate grasping at any theoretical straw that offers even a feeble chance of support in the flood. In this generally febrile condition of our science it is particularly unsafe to draw philosophical conclusions.

When I read books or essays on philosophy that which impresses me most, next to their appearance of getting nowhere, is their apparent attempt to get universal judgments. I realize that much of this impression is probably due to a misconception on my part of the technical terms employed by philosophers. Such words and phrases as " Absolute," " original," " ultimate

---

[1] Read before the Royce Club, Boston, January 18, 1920. The first address to the Club, by its Founder, Professor Josiah Royce, appeared in *Science*, 39, April 17, 1914, pp. 551–566.

ground," "the eternal and immutable," "final causes," "first causes" and "comprehensive view of the universe as a whole" are quite unintelligible to me, although the last which contains "comprehensive," "universe," and "as a whole" seems on its face to be so far emphatic of something that I might reasonably expect to have a precise idea of what that something may be. To me this sort of phrase represents merely emotional aspiration.[2]

I say this in no spirit of contumely; for I shall probably use to-night many technical terms of but poor intelligibility to others—try as I shall to avoid them—and, besides, I suppose that next to the philosopher nobody so insistently tries to reach the "ultimate" as the theoretical physicist!

2. *The Dynamical View of Nature.*—I have spoken of the scientific unrest. The world seems to run in waves of excitement and of hum-drum, of discovery and of codification.

Two hundred years ago Newton set up the fundamental laws of kinetics and of gravitation. Little by little the major part of accurately known Nature came under the régime of these laws. A hundred years ago Laplace had shown the marvelous accuracy with which astronomical observations could be accounted for, and when Ampère and other continental workers came to develop the theory of electric and magnetic phenomena they followed the astronomical model. Stokes and Kelvin were students of dynamics and exponents of the dynamical explanation of Nature.

There was a time some fifty years ago when there was in the minds of many a belief that a dynamical account of the world at large was clearly foreshadowed. Maxwell, to be sure, gave up action at a distance; but, trained as he was in the English school, his point of view was largely dynamical—the dynamics of media, of the special medium known as the ether. He tried, and Kelvin long continued to try, to construct a mechanical model of the ether. There were bizarre theories of matter on a hydrodynamical basis—the atom founded on the persistence of vortices in perfect fluids, the atom built upon the theory of sinks and sources, and finally, not so long since, the general structure raised by Reynolds on the basis of a granular ether.

So happily did phenomena fit into the dynamical theory of

[2] May it be that some philosophers mistake emphasis for precision and description for definition? In times past mathematicians have used terms like "infinite," "infinity," "infinitesimal," "sum of a series," "differential," "imaginary," without sufficient precision in definition with resulting inconclusiveness of proof, fallibility of analysis, and bitter polemic even though confining themselves to much simpler questions than philosophers treat.

Maxwell and so near were we to a satisfactory material basis for the all-pervading medium that not a few of our ablest physicists two or three decades ago felt that we had already entered the hum-drum period of physics where the chief contributions were bound to be merely the more careful determination of physical constants—we were entering upon the reign of the "next significant figure." That there was need for this emphasis on accurate measurement will be granted by all who realize the pitifully inaccurate condition of electrical measurements, the large degree of indetermination in our standard electrical units, which then existed.

3. *The Flood of New Phenomena.*—As a prediction, however, of the future of physical science, the hum-drum forecast was bad. Perhaps the naturalist is too wise to hazard a guess that the future of his science is largely routine. But the outlook of some physicists in the middle nineties was not unlike that of the classifying naturalist who should hold that the thing best worth while was to get some money and some quinine and go in search of a new gnat in South America or a novel Nymphæa in Africa—quite blissfully ignorant of the great new realms of biology that would be opened by Mendel.

In the last twenty-five years we have learned that the atom is not indivisible, but consists of small charged particles or electrons. The physicist likes to have these particles in circular motion; the chemist wants them at rest except for oscillations about a position of equilibrium. We have learned about X-rays —some things about them—and through them we are learning much about many things. We have learned of radioactivity and the self-transmutation of certain elements. We have learned that radiant energy, at least in some of its manifestations, seems to possess a discrete or discontinuous structure, this leading to the perplexities of the quantum theory—perplexities not of physics alone but of mathematics and probability as well. Here is cause enough for unrest.

It may be that to give a satisfactory treatment of quanta we shall have recourse to a belief that space or time or both are themselves discontinuous and that their seeming continuity is but a statistical affair like the continuity of a fluid or solid. There is apparently a long-period oscillation in physics between the continuous and the discontinuous, and there is no reason why in some of its excursions the pendulum should not reach space and time.

4. *The Results of Careful Experiment.*—It is not only the great new things we have learned that trouble us; refinement of

measurement is itself a two-edged sword. Our physical laws are designed to correlate our observations. The correlation may remain good while our measurements are good only to three significant figures and become impossibly bad when we can determine five or six figures.

It has been known for some time that the perihelion of Mercury advances 42″ per century, and Newton's law of gravitation has given no wholly satisfactory account of this advance; there are also some slight unexpected secular variations in the moon's motion—enough perhaps to be visible to the naked eye after some thousand years. So either Newton's law is not perfect or there must be unknown masses, of relatively small amount, circulating in the solar system.

Turning to another branch of physics we have the exceedingly accurate experiment of Michelson and Morley wherein a ray of light is split, part being sent along the direction of the earth's motion to one mirror and back, part being sent an equal distance along the perpendicular direction and back. Now if the light travels in the ether with a velocity dependent only upon the elastic or quasi-elastic properties of the ether, the light going in the direction of the earth's motion and back will, owing to the motion of the mirror, travel a trifle farther than that going in the perpendicular direction. The difference in path amounts under favorable circumstances to about one half of one millionth of one per cent. of the path. This is not much, to be sure, but it should show very plainly in such accurate work as is done in interferometry, and it does not show.

One possible explanation would be that the earth drags the ether with it just as it drags its atmosphere; but there arise serious difficulties when this suggestion is pursued. Another explanation might be that there is a shortening of matter in the direction of motion—and this has been followed up by Lorentz.

5. *The Electric Theory of Matter.*—The suggestion of shortening is not unnatural on the electromagnetic view of matter. When a charged sphere moves, the lines of force, instead of remaining isotropically divergent from the sphere, crowd up toward the equatorial plane perpendicular to the direction of motion and leave the poles of the sphere in the direction of motion with a smaller superficial charge. The tendency of the sphere is to shorten in the direction of motion. Now if all matter is made up of electrons, and if the electrons themselves tend to shorten in the direction of motion, with the attendant alterations in their field of force, it is not unreasonable to sup-

pose that all matter suffers the same shortening when moving through the ether.

The shortening is greater as the velocity of motion is greater. For the earth which moves only 30 km. per sec. in its orbit about the sun, the shortening is only a few inches in 8,000 miles. For an electron which may move at 9 tenths the velocity of light, the shortening is more than half, and if an electron should move at 99 hundredths of the velocity of light, its diameter in the direction of motion would, on this hypothesis, be only one seventh its original amount so that the electron would appear disc-like instead of spherical.

There is another effect of the shortening which is calculable, and that is the gain in electrical energy, and the gain in inertia. A flattened electron when accelerated in the line of its motion tends to become flatter; part of the work done by the applied force must go into crowding the lines of force toward the equatorial plane and hence only part remains over to accelerate the mass. The result is that for a given force the acceleration is less than if the flattening did not take place, or to put it differently, the mass or inertia appears greater. Thus the electron has conceivably two masses or inertias: one due to its mechanical mass, the other to its electrical inertia, and the latter part varies with the velocity in a perfectly definite way. When the experiment of determining the inertia of the moving electron is tried, it is found that apparently all the inertia is that of electrical origin with none left over for ordinary mechanical inertia.

This discovery suggested very strongly that the day for a mechanical explanation of nature has definitely passed and that henceforth, at least for the immediate future, the attempt should be made to give an electrical foundation to mechanics and to all natural phenomena.[3] J. J. Thomson, as a matter of fact, long previously had shown theoretically that a charged material sphere has additional inertia by virtue of its charge, but for ordinary spheres the amount of added electrical inertia is inconsiderable compared with the mass of the material. It was only when the infinitesimal and relatively enormously charged electron became available for direct experimental work that the

[3] The attempt to give a unitary basis to the whole of physical theory has an irresistible fascination and physical theory advances by the strife between this monadic ideal and the ever-growing volume of physical fact. It is like the contest between armor plate and the armor-piercing projectile. Sometimes one appears to be winning, sometimes the other; on the whole, it is a draw. What we have at any time is not one physical theory, but a congeries of theories. See a note by G. W. Stewart, *Science*, 51, 1920, pp. 95–86.

amount of the electrical inertia and the law of its increase with velocity could be determined.

6. *The Old Relativity.*—This idea of shortening in the direction of motion was physically satisfying, but not so philosophically. Space and time are not absolute. Every point of space is just like every other and the only position known or knowable on the basis of Euclidean geometry is relative position. There has been since Newton's time, and before, a natural belief in a principle of relativity. Neither absolute position nor absolute uniform velocity are determinable.

The laws of motion, when resolved far enough, show this. We have to do with accelerations of particles and the value of the acceleration of a point is unchanged when the position of the point is specified by reference to any set of axes whether at rest or moving uniformly parallel to themselves. (The acceleration is affected by a rotation of the axes.) Moreover, the acceleration is not altered by any choice of an initial instant from which to measure time. Finally, as the action and reaction between two particles lie in the line joining them and are supposed to depend only on the distance between the particles, the forces which enter into the equations are expressed in terms of quantities (*i. e.*, distances) which are independent of the choice of axes. Thus arises a belief in relativity.

It is true that the equations of motion for definite bodies moving under the definite conditions inherent in some particular problem may not be analyzed down far enough to reach the simple invariant form indicated. For example, a resistance may be taken as a function of a velocity—a statistical form of expression which gives the gross resultant action of all the myriads of air particles on a bullet. Yet we should believe that if each gaseous molecule were considered, the forces would be representable as functions of the distance; and indeed even when we insert a velocity into the equations of motion it is always regarded as a relative velocity.

We have then in ordinary mechanics a full-fledged, if sometimes quietly ignored, principle of relativity. All that can be treated or known is relative motion when translation is concerned. Newton noticed the difference arising in the case of rotation, which of course involves acceleration even when the angular velocity is uniform.

7. *The Absolute Ether.*—Return now to the ether. This is regarded as a medium filling all space and fixed except for its tremors which show us light. The ether furnishes an absolute

material space, so to speak—and motion becomes conceptually motion relative to the ether. But as no effect of motion of the ether itself (except its tremors about its mean position) has been detected, there is no reason why the ether in its mean position should not be regarded as an absolute and motion relative to it as absolute motion. If this be so, those who desire "absolutes" have their innings.

Here is where the famous Michelson-Morley experiment comes in. If the ether is at rest, pervading all matter but not dragged with it, an interference experiment should clearly show the drift of the earth through the ether—not only the drift due to the motion about the sun, but the drift to the motion of the solar system towards its apex. And the result of the experiment was negative. So also were the results of other experiments which by other means should detect the drift. These negative results may all be explained by shrinkage in the line of motion.

Why then should this effective physical explanation be philosophically unsatisfying? Because. Just because. This is "a woman's reason." It is a good one. I know of none more convincing. Perhaps part of the dissatisfaction is due to our long-continued belief in relativity fortified by a growing distrust of any absolute background which refuses in any way to disclose itself as in motion relative to us.

If lengths in the direction of motion shorten, our measuring instruments shorten so as exactly to counterbalance the abbreviation of the thing measured. Why then should we believe in the shortening at all? May it not be that the shortening itself is only relative? That is, if $A$ and $B$, being in relative motion, observe a distance $PQ$ with meter sticks which are identical when at rest, $A$ and $B$ will differ on the measure of $PQ$ by perhaps one half of one millionth of one per cent.; but why should $B$ claim that it is $A$ who is in motion and who has not corrected for his shortening—why may not $A$ equally well claim that it is $B$ who is in motion and has not corrected for his shortening? And if a third party comes in as referee, and makes a finding different from both because neither is at rest relative to him, what then?

8. *The New Relativity, Space and Time.*—This sort of question did not bother many until Einstein raised it and started out to solve it by boldly affirming that we must construct our space and time concepts in such a way as to avoid these discrepancies and give us a real relativity consonant with physical fact and

independent of the opinions of different observers as to who is and who is not in motion.

A fundamental postulate of his theory is that the velocity of light $c = 3 \times 10^{10}$ cm./sec. appears the same to all observers. This means that velocities do not add according to the ordinary law. Any velocity compounded with the velocity of light gives the velocity of light—no more, no less. Two velocities in the same direction do not compound into the sum of those velocities but into that sum diminished by a small amount.[4] The geometry of velocities, so to speak, is non-Euclidean.

A still more striking revision of ideas is necessary relative to space and time, because these two conceptions become necessarily interrelated, whereas in Newtonian relativity they were independent. Two observers with relative motion measure space differently; they measure time differently; they do not even have the same space and the same time, because the time *or* space measure of either depends on the space *and* time measure of the other, in such a way that they do not agree even as to the simultaneity or not of events happening at different positions— nor do they agree as to the identity or non-identity in position of two events happening at different times.

A fusion of the time and space concepts into a single time-and-space idea is necessary to bring simplicity out of the confusion. What this does to the metaphysician who is interested in settling "what it is that we know and how it is that we know it" is more than I can say. Such discussions as I have read of the categories of space and time would seem to have very little left as an ultimate residuum.

9. *Knowledge of Nature.*—In an article on metaphysics[5] by a philosopher with obvious sympathetic leanings toward Aristotelian methods and an Aristotelian realism I find this qualification in summary: "Aristotle could not know enough, physically, about Nature to understand its matter, or its motions, or its forces; and consequently he fell into the error of supposing, etc." Now relatively speaking there is a good chance that Aristotle in his day did know more, physically, about Nature than a philosopher of to-day can know. There was much less known about Nature then and the sort of thing known was much simpler, and Aristotle was a profound student of Nature with

---

[4] If $B$'s velocity appears to $A$ as $u$ and $C$'s velocity appears to $B$ as $v$, then the composition of the velocities is by definition that velocity which $C$ appears to have to $A$. In ordinary mechanics this is $u + v$, in relativist mechanics it is not.

[5] See "Metaphysics," Encyc. Brit., 11th ed., Vol. XVIII.

the best resources of his time largely at his disposal. Aristotle probably knew Nature better than a physicist to-day knows physics or a chemist chemistry.

Here again I must state that I am not trying to be contumacious toward the philosophers in this group. I appreciate thoroughly that our Founder tried to collect into this aggregation students of many a field for the very purpose of pooling our philosophical interests and our scientific information of mutual philosophic import. I wish to emphasize merely this: That nobody at any time can know enough about Nature to understand its matter, or its motions, or its forces and that consequently everybody will fall into error if he goes to supposing this or that or anything else which is of the sort that later discoveries may upset.

Certainly it is difficult to ascertain what it is that can not by any future development be upset—and as I understand them this it is, and this alone, that interests many philosophers. With Einstein's relativity of 1905 space and time as separate things disappeared into the space-time fusion. Does everybody believe this? By no means. The great majority are still either indifferent to it, because it does not immediately affect their work, or set against it because it is not "physical" but "philosophical."

10. *Force, Real or Fictitious.*—Query: When is a force not a force? Answer: When it is a "reversed effective force." To amplify this conundrum I will recall to your mind the general principle of d'Alembert to the effect that: The impressed forces acting on a body taken with the reversed effective forces form a system in equilibrium. This amounts really to transposing the terms in the equation $Ma = F$ so that $0 = F - Ma$. The force $F$ is the impressed force, the force $- Ma$, the negative of the mass times the acceleration, is by definition the reversed effective force.

This principle is of great convenience. It enables the engineer to treat a problem of motion in a curve by the introduction of the fictitious "centrifugal force," which is of course the reversed effective force—not a true physical force (the physical force is centripetal). By this fiction the force-analysis relative to the body in curvilinear motion is reduced to a simple static diagram.

Suppose we are in a uniformly moving train and cannot look out. Thanks to the principle of relativity, all goes nicely according to the Newtonian laws, until we strike a curve. Then everything has an outward acceleration—apparently. Really there is

no outward acceleration. The bodies are merely trying to maintain themselves in uniform motion in a straight line—at least this is the interpretation we have come to put on the phenomenon since Newton's time.

Now as a matter of fact we are on a very smoothly running train all our life; but the earth turns with so small an angular velocity that it has been only in recent times that it has become convenient, and consequently real, among scientists, to regard the world as turning. And only in still more recent times have refined experiments such as those with Foucault's pendulum and with gyrostats shown us before our very eyes this rotation of our frame of reference. We may regard the earth as non-rotating if we choose, but the introduction of "centrifugal" and "Coriolis" forces which are fictitious reversed effective forces. Indeed, for the solution of such problems the introduction of these fictitious forces is generally more convenient than to use a fixed frame of reference, even though we know better than to adopt as a philosophy a non-rotating earth and as physically existing the fictitious forces in question. We are very wise about things we have long since come to believe.

But just suppose that somebody tells us that the force of gravity is physically non-existing quite as much as the centrifugal or Coriolis force, and that the reason we think that gravity is real is essentially the same that leads the untutored mind to believe there is a physical force acting to move objects to one side when a train goes around a curve—namely, an unhappily ignorant view of Nature. This is what Einstein asserts.

11. *The Space-Time Path.*—He goes further and maintains that instead of every particle of matter attracting every other particle according to the Newtonian law, each particle goes its way on the shortest or straightest path possible, when both time and space are considered in a single space-time manifold.

That the path of the planets is really nearly straight may easily be seen. In the motion about the sun the earth turns through about 1 degree per day and departs from its recti-linear motion by 22,000 kilometers in 2,600,000 when space alone is considered. But, using the velocity of light as a standard, 1 second of time is equal to 300,000 kilometers of distance. Hence in a day, which is 86,400 seconds, the advance in time is the equivalent of some 26 billion kilometers, while the motion in the path is only one ten thousandth as much and the motion across the path is only 22,000 kilometers or less than one millionth as much.

In the space path we have a departure per day from rectilinear motion of about one part in a hundred; on the space-time path the departure from rectilinear motion (motion with uniform velocity) is about one part in a million, which represents a much smaller curvature—the path is nearly straight.

This may all be made clearer by regarding the space-time path as constructed in the following manner. Draw from the sun perpendicular to the plane of the earth's orbit a line which shall represent the time-axis and disregard the third spatial dimension. Now for each km. that the earth moves around in its orbit, it must be considered to move in time by 10,000 km. The path of the earth in space and time on this diagram is therefore a helix with an extremely steep pitch winding once per year about the cylinder standing in the earth's orbit but advancing ten thousand billion km. while "circulating" one billion km.

Such a helix departs very little from a generator (vertical element) of the cylinder and is nearly straight. Not much quantitative change in our space-time measurements, however great the qualitative change might be in our space-time concept, might make the space-time path a geodesic or shortest path in a curve space-time manifold. Einstein shows how to make the change.

The mathematics is complicated; it depends on the theory of quadratic differential forms which to me seems perhaps the most intricate branch of pure mathematics and is at all events one unfortunately little studied in America—or anywhere else except in Italy. Einstein himself stumbled many times before he succeeded, not without the aid of others, in making the change he desired, but his progress toward his objective was relentless and must have required both a keen imagination and an iron will.

12. *"Curved" Space.*—A word about curved space. This is a difficult concept to non-mathematicians. All will admit that a two-dimensional spherical surface is curved in three dimensions. The difficulty is in seeing how a person limited to two dimensions in his motions and without any imagination of a third dimension could conclude from his observations on the spherical surface that the surface was not flat but curved. Or how we, limited to three dimensions, could by observations in three dimensions decide that our space was curved. Or, still worse, how we, limited to four dimensions of space and time, could by observation determine that our space-time manifold was curved.

Let us consider the case of the spherical surface and compare it with that of the plane. Suppose the plane inhabitant selects an origin $O$ and a line $OL$ issuing from it and specifies the portion of a point $P$ by the distance $OP$ of $P$ from $O$ and the angle $POL$ giving the "bearing" of $P$ relative to $OL$ (polar coordinates). Then he will find that the distance between two nearby radial lines $OP$ and $OP'$ at equal distances $OP = s$ from the origin $O$ will be the product of $OP = s$ and the infinitesimal angle $POP'$, and that the length of the circle about $O$ as center and with $OP$ as radius is $2\pi OP = 2\pi s$.

Now the inhabitant of the sphere will find a very different result by the similar procedure. He selects his origin $O$ and line of zero "bearing" $OL$. (This line will from our external point of view be a great circle, but from his point of view a straight line or geodesic, because its curvature being wholly normal to his space is not directly perceptible.) He will find the infinitesimal distance at any point between two nearby lines $OP$ and $OP'$ not as $OP$ times the angle $POP'$ but as this product multiplied by $\sin s/a \div s/a$ and the whole periphery of the circle equidistant from $O$ not as $2\pi OP$ but as $2\pi OP$ multiplied by the same factor, where $a$ is some constant.

The difference between the results

$$2\pi s \quad \text{and} \quad 2\pi s \times (\sin s/a \div s/a)$$

for the perimeter of circles of radius $OP = s$ is surely significant of something. You may call it significant of the curvature of the sphere even though you attach no other significance to the word curvature. It is indeed probable that the use of the word curvature here is attributable to our three-dimensional view of the sphere.

When we work in our flat three dimensions we find the area of a spherical surface of radius $OP$ to be $4\pi \overline{OP}^2$; if we should find it to be something else such as $4\pi \overline{OP}^2 \times (\sin s/a \div s/a)^2$ we should attribute this to some property of our space and seek to interpret the constant $a$ relative to the space. We may or may not speak of our space as curved but we do at least recognize that the formula for the area of a sphere of radius $OP$ is not directly proportional to the square of $OP$ when $OP$ is large though it is when $OP$ is infinitesimal relative to the constant $a$.

In the Newtonian relativity space was flat and independent of time; in the Einstein relativity of 1905 space and time had to be considered together but the manifold thus obtained was still flat and no special account was taken of gravitation, the theory was essentially electromagnetic in the sense that it was fitted to

the electrical conception of Nature; in the Einstein new rela-
tivity space and time must still be considered together but the
space-time manifold has become curved and the curvature has
been so adjusted that the major effects of gravitation, as ordi-
narily considered, have been accounted for by the curvature
of the space-time manifold so that every particle of matter pur-
sues a geodesic or straightest path in the four-dimensional
manifold.

The further explanation of detail is complicated by the fact
that there are different sorts of curvature. Consider again the
spherical surface. There is one curvature, called the mean
curvature, which is estimated relative to the radius and is ex-
actly equal to the reciprocal of the radius, $1/a$. There is another
curvature, called the total curvature, which is estimated rela-
tive to the square of the radius or to the area and is taken as
equal to the reciprocal of the square of the radius, $1/a^2$. On
the sphere both curvatures are constant; in the plane both are
zero. On a cylindrical surface of radius $a$, the mean curvature
is $1/2a$, the total curvature is zero, both are constant.

An interesting class of surfaces is that formed by soap films
stretched across various forms of wire. The film has to stick to
the peripheral wire, but is otherwise free to contract under the
influence of the surface tension of the film. The result is that
the film shrinks into the form of the surface of least area, the
so-called minimal surface, which can span the periphery. Such
surfaces have their mean curvature constantly equal to zero.
They are saddle shaped with as much positive curvature in one
normal section as there is negative curvature in the perpen-
dicular section; the total curvature of such surfaces is negative
and variable, being zero at the periphery where the film is at-
tached to a segment of straight wire. In stepping up to three
dimensions there are curvatures expressible as reciprocals of
lengths, or of areas, or of volumes, and of these the first two
are types of mean curvature, the last of total curvature.

Generalizations to four dimensions may be made. But when
we have to deal with gravitational effects of matter considered
to be at rest, as is the case with the dynamics of the solar sys-
tem so far as these are attributable to a sun regarded as at rest
(and in abstraction from all perturbative effects) it has been
shown by Levi-Civita that space and time may be separated (as
is also the case in the 1905 relativity where observers agree as
to what is at rest) so that phenomena pass in time and in space.

The three-dimensional space surrounding the central sun is,
however, curved, but with zero mean curvature much as the

soap film is curved but with no mean curvature. Within the mass of the sun the mean curvature is not zero, but is intimately connected with the density of matter or of energy.[6] For Einstein's theory is one of energetics.

13. *Energy and Mass.*—We have stated the theory chiefly with relation to its modification of our space and time concepts and with but little account of its underlying physics or philosophy except in so far as concerns the early relativity of 1905. It is necessary to return to the physics and philosophy of Einstein between 1905 and 1915.

It was seen that the mass or inertia of any electron was wholly electrical. Now mass according to our older concepts has two properties—inertia and weight, for both of which Newton set up the quantitative formulation. Does the electron weigh? Is the electrical inertia, which is proportional to the electrical energy, of the electron subject to the law of gravitation? If matter is wholly electrical, the simplest hypothesis is that the electron does weigh.

A beam of light represents energy; according to Maxwell it represents also momentum which should be disclosed as the pressure of light when the light is either absorbed or reflected. The pressure has been measured. Light, therefore, has the two principal characteristics of moving matter, energy and momentum, why should it not have the other two, inertia and gravitation?

The physicist generalizes on very slight evidence if his fancy, his esthetic sense, is sufficiently insistent. We have come to believe[7] that every quantity of energy, of whatever sort, represents just so much mass, that every amount of mass represents just so much energy of some sort, and that every bit of energy attracts every other bit of energy just as much as every particle of matter attracts every other particle—which on the new theory is really not at all, so that we must state it other-

---

[6] That Einstein's gravitational theory makes it *necessary* for us to think of our space-time manifold as curved I would not say. The non-Euclidean plane geometry of Lobatchevsky may be developed without specific reference to the curvature of the Lobatchevskian plane; or it may be treated as the geometry on a surface of constant negative total curvature such as the pseudosphere; or, by the aid of the Cayleyan system of measurement, as a type of geometry inside an ellipse or circle in the Euclidean flat plane. See " Geometry," Encyc. Brit., 11th ed., Vol. XI.

[7] I am here trying merely to state Einstein's ideas as I understand them, as throughout this paper, without wishing to give the impression that all of these ideas are yet universally accepted or even certain to become widely accepted. The theory is still young and may succumb to infant diseases instead of to general disintegration after a ripe old age.

wise, namely, that every particle of energy puts its appropriate crimp into our space-time manifold. Or to give a better phrasing, departures of our space-time manifold from flatness are correlated definitely with what we otherwise term energy.

The numerical relation between mass in grams and energy in ergs is this: The mass is the energy divided by the square of the velocity of light. Thus one gram of matter represents $9 \times 10^{20}$ ergs or over 21 billion calories. Conversely, a hot brick is heavier than a cold one—it has more mass by the amount of heat added if measured in ergs and divided by $9 \times 10^{20}$. To heat 1 kg. of water 100° C. requires about $4.2 \times 10^{12}$ ergs; the increase in mass or weight is therefore only about 5 millionths of a milligram. If there were any way of making even a small part of the energy latent in matter available, our supply of useful energy would be enormously increased. It may be that the sun is in a condition where such conversion is going on. Radioactive matter is in such a condition.

14. *Einsteinian Physics.*—But to return now to light regarded as suffering attraction because, being energy, it is mass. A simple calculation of the deviation of a ray of starlight passing close to the sun on its way to our eye shows that on the Newtonian theory the deviation would be 0.87″ and would fall off inversely as the distance of the periphelion from the center of the sun. (In the astronomical work of determining parallaxes, 0.87″ is a very large quantity, larger than any stellar parallax.) This was Einstein's first estimate. When he had formulated his complete theory, a new calculation showed that, on the curved space-time hypothesis, the deflection should be twice as much, namely, 1.75″.

The solar eclipse of last spring gave an opportunity to measure the deflection which was found to be in satisfactory accord with the new theory, not with the ordinary Newtonian. The report of these findings as presented at a recent meeting of the Royal Society was the occasion for the newspaper interest in the obsolescence of Newton.

Another fact that was really the cause of an early widespread interest in Einstein's work on the part of astronomers was that from his theory a calculation of the motion of Mercury showed an advance of the periphelion of 42″ per century, the amount that had so long defied satisfactory explanation.

A third consequence of the theory is that light originating near a great mass such as the sun should be of lower frequency than if originating near a small mass like the earth; the lines in

the sun's spectrum when compared with laboratory standards should be shifted toward the red by a minute amount much as if the sun were traveling away from the earth.

This prediction has been subjected to a most searching experimental analysis at Mt. Wilson and remains unverified, though if the effect existed, it ought to have been found. The phenomenon may ultimately be found or its absence satisfactorily explained. On the other hand the deflection of rays of light by the sun, as solar eclipses are again searched, may be found to be absent or may be attributable to refraction in an extremely tenuous solar upper atmosphere.

There are, moreover, some cosmographical speculations by Einstein relative to the finiteness of our sidereal universe, and the distribution of matter in it, which I should mention if there were time. Cosmography, however, has become a large science in itself, in the last few years, with tolerably definite and highly interesting conclusions which should be treated in their entirety, and to this whole the present contributions of Einstein's theory are, of course, small.

15. *Einsteinian Philosophy.*[8]—Our interest here is primarily with the philosophical and there is one fundamental view of Einstein's which seems to have been his guiding star through his earlier and his later relativity. In 1905 he desired to construct a theory independent of the rest or state of uniform motion of any observer—certainly a philosophical ideal.

Later he introduced his "postulate of equivalence" wherein he states that from observations in a closed system (our railroad train) an observer should be unable to tell whether he were at rest in a gravitational field or in motion with a uniform acceleration. Thus not only can the isolated observer not determine his absolute motion (velocity), he can not tell how much of the force he feels may be due to attracting masses and how much of it to his own acceleration. This is a reasonable philosophic idea or physical assumption.

We all know that we feel lighter in an elevator moving with a downward acceleration whether this be in starting downward or in stopping toward the top of a rise. And we have all heard the more complicated conditions arising in flight, and the indetermination which arises thereunder, discussed by aviators when telling of flights in clouds.

One can see in the equivalence postulate the germ of the

idea that gravitation is a fictitious force, and the possibility that an observer by his own measurements would come to some space-time conception which in itself should cover the phenomena ordinarily attributed to gravitating masses.

Another proposition which guided Einstein was that the laws of nature, whatever they be, should be expressible in such a manner as to remain valid when any system whatsoever of time and space coordinates be used. This insistence on a general invariantive or covariantive statement of law is responsible for much of the mathematical difficulty in the formulation of the theory, but we have the author's testimony that it was of considerable heuristic value.

The relativity principle whether old or new (1905) regarding velocity and the equivalence postulate regarding gravitation and acceleration are types of a " philosophy of ignorance " such as is often met in the theory of probability, in the " principle of sufficient reason," in arguments from symmetry, etc. It does valiant service for the wise, but is a dangerous tool for the really ignorant.

This method in science may be called the philosophic if it be desired to have a phrase contrapuntal with the scientific method in philosophy[9] concerning which Mr. Hoernle spoke to us at our last meeting. The meaning of the adjective is, however, not positive but comparative and reaches out toward the superlative in either case only as the adjectives philosophic and scientific are conceived in their narrowest senses.[10]

16. *Man's Place in Nature.*—Man's place in physical Nature used to be central. His earth stood still and everything re-

[9] The method, roughly, of Russell and Whitehead. Undoubtedly life is too short to cast everything into a rigorous mould and much would be lost by an insistence on the method. In mathematics itself the extreme rigorous and logical method is not pursued except in restricted fields or when dangerously critical problems are attacked. But a well-trained mathematician or mathematical physicist must to-day have had some education in extreme rigorous analysis that he may come to know the pitfalls that are in the way of careless workers and avoid them. A similar training in the scientific method for its educational value should be required of students of philosophy; it saves time in the long run.

[10] Although Einstein's method has been called philosophical, one should not infer that Einstein is technically a philosopher or that philosophical technicians will agree as to the system of philosophy that is fortified by his conception of Nature. Mr. Hoernle has called my attention to some spirited correspondence between Whitehead and Walker in the *Nation* (London) for Nov. 15, 29 and Dec. 13, 27 (1919) wherein the former claims that Berkeley has been avenged and the Aristoteleans put to rout, while the latter finds that only now has Aristotle completely come into his own. Alas, poor Yorick!

volved around it and him, and his god or gods sat off where they were put and were pleased by his frankincense. And he saw everything he had made and behold, it was very good, and when he had thus done and been satisfied with the goodness thereof he rested,—upon his Carnegie pension, or otherwise.

How near to this point of view some metaphysical idealists may still be I do not venture to surmise. Certainly the perusal of some presentations of the more extreme idealism gives the scientist the impression that there have been those who would convert that reverent and realistic scientific lullaby

> Twinkle, twinkle, little star;
> How I wonder what you are
> Up above the world so high
> Like a diamond in the sky.

into some such blasphemous conceit as

> Twinkle, twinkle, little star;
> I know what you really are—
> Just a greedy summer fly,
> Creature of my inner eye.
> Nibbling at that hunk of cheese
> Half a mile beyond the trees.

Probably none of their ilk remain.

Man's place to-day in physical Nature is far from central. He should be decidedly humble. He knows infinitely little and what knowledge he has is for the most part either a partial understanding of discrete facts or a conventional correlation of different facts based not upon ultimate truth but upon the brief convenience of the leading minds of his time,—to the lesser minds the convenience of the leaders may be a serious inconvenience.

In Homeric times the earth was flat. A thousand years later it was round, first by philosophic or mystic fancy, then by indisputable proofs of Aristotle, measured with accuracy by Eratosthenes and charted by Ptolemy. For a thousand years under Christianity the earth was again flat, and no "good" man could think otherwise. Since a bare five hundred years the world has been round again, first for martyrs and then for all. Who shall say that it will not again become flat? Viewing history and the prehistoric record as a whole who dares predict that some barbarian horde, some incendiaries of libraries, some religious bigotry will never wipe out present knowledge, replacing it for long periods of time by some earlier doctrine that we now deem less convenient, untrue.

Aristarchus had a good heliocentric theory of the world;

why should Hipparchus have chosen the geocentric—the greater reversing the lesser scientist? Why should Ptolemy have continued the error? What seemingly sound scientific doctrines of to-day are the geocentric systems of Hipparchus and Ptolemy, overriding, reversing the saner positions of some humble Aristarchus of a generation ago who may yet wait centuries for vindication? May it be the luminiferous ether? Einstein is quoted as saying there is no ether. Is the early corpuscular optics to return, modified of course sufficiently to account for our present continuous optics, and shaped to cover our discontinuous optics of quanta? I do not know.

The absolute and the universal, though perhaps a guiding inspiration, remain mere emotional aspirations. Under the analysis of the last century even our logic broke down before the paradox of "the class of all classes," and however much we may push off this contradiction by finer analysis, I do not believe we shall be rid of it until we abandon such phrases as the class of all classes,—any more than the mathematician was rid of his difficulties with the simplest type of infinity until he insisted that this was no infinite except as an indefinitely increasing finite, so that the static infinite became the kinetic finite.

We are acquiring and discarding, learning and forgetting, and at any time there is nearly an equilibrium between import and outgo; for some centuries we gain, for others we decline with a fall far more swift and precipitous than the arduous climb.

Not the survey of the latest theories, nor estimates of the immediate future, nor yet the study of the past few centuries will suffice to ground a philosophy. Several millenniums are necessary. On the whole we seem to be getting on. At least we must have faith to carry on.

And so for a philosophy I turn back to the first of the metaphysicians whose doctrine, as I understand it, may be phrased in the words of the passing song, "We don't know where we're going, but we're on our way." Far better so than "All dressed up and nowhere to go"—yes, worse, left behind, in the majestic march of Nature and of man's living thought upon it.